Tri-Level Optimization of Regional Water Supply System

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1. ARIZONA WATER ISSUE

Nowhere in the United States is the projected imbalance between water supply and demand greater than in the Colorado River Basin. The Bureau of Reclamation (BOR) Colorado River Basin Study [1, 2], the first of its kind, focuses upon the entire basin, with imbalance in supply and demand projected to be over 3.2 million acre-feet by 2060. The implications of water shortages are most pressing for Arizona and, in particular, central Arizona. The compromise leading to federal construction of the Central Arizona Project (CAP) requires the recipients of CAP water, including the Phoenix and Tucson metropolitan areas to accept water rights that are junior to other deliveries of Colorado River water. As a consequence, the CAP supply will be the first to be reduced in times of shortage. The Arizona Dept. of Water Resources’ (ADWR) Strategic Vision for Water Supply Sustainability [3] concludes that Arizona could face an annual water supply imbalance in the next decades of about 1 million acre-feet (nearly 15% of the total available supply).

The imbalance between water supply demands would generate serious impact on the cities and towns in Arizona, and Tucson not an exception. Therefore, identification and development of water supply planning and management for urban areas is imperative for the sustainability and long term economic health of the state. To that end, several studies have used mathematical optimization models to determine the optimal strategies for using given water resources in the most economical way. Almost all current models, however, assume that the system facilities, such as booster pump stations and water treatment plants, will always function as desired. In reality facilities can fail caused by the uncertain variation in system capacity, aging of system, or natural/manmade hazards. The question then becomes, what are the best strategies to meet water demands over time for municipal, agricultural, and industrial communities in the presence of uncertain facility failures.

To answer this question, this study has applied a method developed by Lan et al. [4] with more realistic facility failure scenarios to make a decision on storage tank building timing, location, size, and flow allocation of a water supply system. As a result, the advanced method necessary to solve complex multi-level optimization problems is developed and applied. A significant advantage of the proposed scenario-planning based approach is its ability to incorporate social and institutional uncertainties and judgment. In the proposed study, the framework is applied to regional-scale water system located at the southeastern Tucson.

2. METHODOLOGY

2.1. Tri-Level Optimization Model

In this section, tri-level optimization model is presented for RWSS planning considering the risk of facility failure (Figure 1). The model is mixed integer linear programming (MILP). The complete notation used in the deterministic model and the rest of this report is listed in Table 1.

2.1.1. Objective Function

Model formulation is developed by in collaboration with Dr. Nan Fang (assistant professor in the department of system and industrial engineering). The objective function (1) consists of storage tank capital cost (first sum), normal operation cost (second sum), operation cost under failure scenario (third sum), and minimization of water shortages (last term). Constraints for the objective function (1) are explained in next 3 subsections.

\[
\text{Minimize } \sum_{i \in S} \sum_{k \in K} c_{ik} x_{ik} + \sum_{(i,j) \in A} \sum_{t \in T} c_{ijt} q_{ijt}^o + \sum_{(i,j) \in A} \sum_{t \in T} c_{ijt} q_{ijt}^f + \sum_{i \in U} \mu_{itf} \theta_{itf} (1)
\]
2.1.2. Optimal System Design
The objective function in (2) is a sum of storage tank design costs. Constraints (3) and (4) impose restrictions on the design decisions. Constraint (3) enforces the fact that at most one candidate capacity may be selected as the size of a storage tank capacity. Constraint (4) enforces the binary requirement.

Minimize $\sum_{i \in ST} \sum_{k \in K} c^c_{ik} x_{ik}$ \hspace{1cm} (2)

Subject to

$\sum_{k \in K} x_{ik} \leq 1, \hspace{1cm} i \in ST$ \hspace{1cm} (3)

$x_{ik} \in \{0,1\}, \hspace{1cm} i \in ST, k \in K$ \hspace{1cm} (4)

2.1.3. Optimal Water Allocation at Normal Operation
Here, the purpose of objective function (5) is to minimize the cost allocation (the total cost to treat and distribute flow in the system) to all sources at a normal operation.

Minimize $\sum_{(i,j) \in A} \sum_{t \in T} c^o_{ijt} q^o_{ijt}$ \hspace{1cm} (5)

Subject to

$- \sum_{(j,(i,j) \in A)} q^o_{ijt} + \sum_{(j,(j,i) \in A)} \mu_{ji} q^o_{jit} = b^o_{it}, \hspace{1cm} i \in NS; t \in T$ \hspace{1cm} (6)

$W^o_{it-1} + \sum_{(j,(i,j) \in A)} \mu_{ji} q^o_{jit} - \sum_{(j,(j,i) \in A)} q^o_{jit} = W^o_{it}, \hspace{1cm} i \in S, t \in T$ \hspace{1cm} (7)

$V^o_{it-1} + \sum_{(j,(i,j) \in A)} \mu_{ji} q^o_{jit} - \sum_{(j,(j,i) \in A)} q^o_{jit} = V^o_{it}, \hspace{1cm} i \in ST, t \in T$ \hspace{1cm} (8)

$\sum_{(j,(i,j) \in A)} q^o_{ijt} \leq P^o_{it}, \hspace{1cm} i \in N, t \in T$ \hspace{1cm} (9)

$\sum_{(j,(j,i) \in A)} \mu_{ji} q^o_{jit} \leq RT^o_{it}, \hspace{1cm} i \in N, t \in T$ \hspace{1cm} (10)

$\sum_{(j,(i,j) \in A)} \mu_{ji} q^o_{jit} \leq \sum_{k \in K} c^c_{ik} x_{ik} |_{i \in ST}, \hspace{1cm} i \in N, t \in T$ \hspace{1cm} (11)

$0 \leq q^o_{ijt} \leq Q^o_{ijt}, \hspace{1cm} (i,j) \in A, t \in T$ \hspace{1cm} (12)

$0 \leq W^o_{it}, \hspace{1cm} i \in S$ \hspace{1cm} (13)

$0 \leq V^o_{it}, \hspace{1cm} i \in ST, t \in T$ \hspace{1cm} (14)

Each node $i \in NS$ is associated with quantity $b^o_i$ that represents the net water supply or demand rate. The variable $b^o_i$ is negative, positive, and zero for a supply node, demand node, and transit node, respectively. The variable $W^o_{it}, i \in S$ denotes aquifer storage for node $i$ if node $i \in S$. $V^o_{it}, i \in ST$ means tank storage for node $i$ if node $i \in ST$. $P^o_{it}, i \in N$ denotes pumping capacity for node $i$ and $RT^o_{it}$ represents the treatment capacity if $i$ is a treatment plant or recharge facility. For each arc $(i,j) \in A$, $Q^o_{ijt}$ the pipeline capacity, $\mu_{ij}$ denotes the loss multiplier to account for water leakage.
The decision variables $q_{ijt}^f$ denotes the flow from node $i$ through node $j$ at time $t$ and $c_{jit}$ is the unit cost for carrying flow from node $i$ through node $j$ at time $t$.

The objective function (5) means the total operational cost over normal operation time. Constraints (6), (7), and (8) represent conservation of mass for non-storage, storage nodes, and tanks respectively. Constraints (9), (10), and (11) are the capacity constraints limiting the total flow that may enter and/or leave certain nodes. Constraint (11) bounds the total flow into a node by its storage tank. Constraint (12) give bounding restrictions for the flow variables. Constraints (13) and (14) give bounding restrictions for the capacity of storage and tank nodes.

2.1.4. Optimal Water Flow Allocation under Failure Scenario

Here, objective (15) is to minimize the water shortage to all users under the failure scenario. In the objective function, $\sigma_{it}$ denotes penalty cost for water shortage. Note that the constraints in this section are almost the same constraints for optimal water allocation at normal operation. To take into account pipe failure, $\delta_{ijt}^f$ (25) is created. The variable $\delta_{ijt}^f$ is a binary variable for arc $(i,j)$ failure. If arc $(i,j)$ is failed, then value of 1 is assigned to $\delta_{ijt}^f$, 0 otherwise.

\[
\begin{align*}
\text{Minimize} & \quad \sum_{(i,j) \in A} \sum_{t \in T_f} c_{ijt} q_{ijt}^f + \sum_{i \in U} \sigma_{it} \theta_{it}^f \\
\text{Subject to} & \quad \sum_{(j,i) \in A} q_{ijt}^f - \sum_{(j,i) \in A} \mu_{ji} q_{jit}^f - \sum_{(j,i) \in A} q_{ijt}^f = b_{it}, \quad i \in NS; \ t \in T_f \\
& \quad W_{it}^f + \sum_{(j,i) \in A} \mu_{ji} q_{jit}^f - \sum_{(j,i) \in A} q_{ijt}^f = W_{it}^f, \quad i \in S, t \in T_f \\
& \quad V_{it}^f + \sum_{(j,i) \in A} \mu_{ji} q_{jit}^f - \sum_{(j,i) \in A} q_{ijt}^f = V_{it}^f, \quad i \in ST, t \in T_f \\
& \quad \sum_{(j,i) \in A} q_{ijt}^f \leq P_{it}, \quad i \in N, t \in T_f \\
& \quad \sum_{(j,i) \in A} \mu_{ji} q_{jit}^f \leq RT_{it}, \quad i \in N, t \in T_f \\
& \quad \sum_{(j,i) \in A} \mu_{ji} q_{jit}^f \leq \sum_{k \in K} C_k x_{ik} |_{i \in ST}, \quad i \in N, t \in T_f \\
& \quad 0 \leq q_{ijt}^f \leq Q_{ijt} (1 - \delta_{ijt}^f), \quad (i,j) \in A, t \in T_f \\
& \quad 0 \leq W_{ijt}^f, \quad i \in S, t \in T_f \\
& \quad 0 \leq V_{ijt}^f \leq C_k x_{ik}, \quad i \in ST, t \in T_f \\
& \quad \delta_{ijt}^f \in \{0,1\}, \quad \delta_{ijt}^f \begin{cases} 1 & \text{if } (i,j) \text{ is failed} \\ 0 & \text{otherwise} \end{cases} \quad (i,j) \in A, t \in T_f
\end{align*}
\]

3. APPLICATION

3.1. Study Area

The network flow model consists of surface water supply, water/wastewater treatment plants, recharge facilities, reservoirs, a well field, as well as municipal, agricultural, and industrial users. The study area is approximately 276 square miles and composed of 19 pressure zones separated.
by 110 ft in elevation (Figure 2) denoted in alphabetical order starting with Zone C. The network flow model computes the optimal storage tank construction timing and sizing that provides the storage tank capital cost and allocation of potable and non-potable water in a manner that minimizes the operational cost under normal and failure scenario operations while minimizing water shortages. The model is solved using CPLEX [5] with MATLAB [6].

Multi-year population estimates were taken from the Water & Wastewater Infrastructure, Supply & Planning Study (WISP, 2009). Since the majority of population increase is expected in the lowest 7 pressure zones (Zones C to I), only these zones were modeled with demands from Zone J and higher added to Zone I’s water consumption. From available demographic studies, the RESIN area population will increase from 33,300 to as many as 372,000 people over the planning period. Demands are calculated using Tucson Water’s per capita usage of 135 gallon per capita per day (GPCD) for municipal and commercial areas. RESIN area receives 130,000 acre feet of CAP water annually.

Wastewater returns are assumed to be a percentage of the potable use from municipal and industrial demand nodes. Fifty-two percent of the total demand is assumed to be for potable uses with the remainder non-potable consumption. These percentages are based on 20 GPM for commercial outdoor uses, 70 GPM and 45 GPM for household indoor use and outdoor consumption, respectively. Outdoor water use and irrigation are assumed to be consumptive uses. Potable water sources can supply both potable and non-potable user demands depending upon water availability and cost. Non-potable supplies are only acceptable for non-potable uses. Figure 3 shows the network configuration. A detailed explanation on network flow dynamics can be found in [7].

3.1. Unit Cost

Unit costs are either given from local water officials at Tucson Water and Pima County Wastewater or approximated using the general horsepower pump equation as:

\[ P_{HP} = H \frac{Q \cdot f_{peak}}{3960 \cdot e_{pump} \cdot e_{motor}} \]  

In Equation (26), \( H \) is head supplied by pump in feet; \( f_{peak} \) is peaking factor; \( Q \) is flow in gallon per minute; \( e_{motor} \) is pump efficiency; \( e_{pump} \) is pump hydraulic efficiency; \( P_{hp} \) is required power for peak flow in horse power. For this study, \( f_{peak} \) is assumed to be 1 while both \( e_{motor} \) and \( e_{pump} \) are 90%. The energy consumption is multiplied by the unit energy cost per kwh to determine total energy cost of which value is 0.08.

In addition to pumping costs, treatment and/or operations and maintenance costs are also added to energy costs to compute the total cost for each arc. The latter costs are either approximated from treatment plants with similar capacities as reported by the EPA or by local officials. A 4% discount rate was applied to the unit costs to compute the present worth of the operation costs.

Tank capital cost is determined based on equation (27):

\[ c_{ik}^c = V_{storage} \times c_{tank} \]  

(27)
where \(c_{ik}^C\) is the capital cost for building a storage tank with capacity \(C_k\) at node \(i\), \(V_{storage}\) is the volume of tank in gallon, \(c_{tank}\) is the cost of storage tank capacity ($/gallon). For this study, \(c_{tank}\) value of 0.80 $/gallon is used based on engineering judgment.

3. 2. Failure Scenario

The failure scenario is determined based on the risk-based failure mode effects mode and criticality analysis on the network [7]. The analysis revealed that Pipe C failure is the most critical, and it causes the most serious water shortage. The location of the failure is represented by a cross in Figure 3. For this study, Pipe C failure occurs in June 1st, 2020, 2030, and 2040.

4. KEY FINDINGS

Total costs as determined by tri-level optimization is shown in Figure 4 as a bar graph. In the bar graph, tank construction and operation costs represented by blue and orange bars, respectively. The total costs is approximately 115.2 million dollars. Total tank capital cost optimization is 3.2 million dollars and total operation cost is 112 million dollars. The capital cost of tanks is much smaller than the operation cost since the tank construction is one time process while the operation is ongoing process for 41 years.

The locations and sizes of tanks are shown in Table 2. The largest tank (1 MG) is constructed in Zone HS. Second largest tank is in Zone C (0.6 MG). Based on the results from the optimization the other pressure zones contain 0.4 MG tanks.

Future effort should modify the tri-level optimization model that optimize multistage infrastructure/tank construction under Pipe C failure scenario as this study only considered immediate infrastructure build out. Furthermore, the model efficiency can be enhanced by incorporating the Benders decomposition [8].

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REFERENCES

### Table 1. Variable nomenclature

<table>
<thead>
<tr>
<th>Indices and sets</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Set of nodes $i$ in the water supply system, $i \in N$</td>
</tr>
<tr>
<td>$ST$</td>
<td>Set of storage tank nodes, $ST \subseteq N$</td>
</tr>
<tr>
<td>$S$</td>
<td>Set of non-storage nodes, $NS \subseteq N$</td>
</tr>
<tr>
<td>$U$</td>
<td>Set of user nodes, $U \subseteq N$</td>
</tr>
<tr>
<td>$K$</td>
<td>Index set of candidate capacity for tank, $k \in K$</td>
</tr>
<tr>
<td>$A$</td>
<td>Set of arcs $i, j$</td>
</tr>
<tr>
<td>$T$</td>
<td>Set of time periods, $t \in T$</td>
</tr>
<tr>
<td>$\mathcal{T}_f$</td>
<td>Set of time period when some arcs fails.</td>
</tr>
<tr>
<td>${t_f}$</td>
<td>Failure period</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_k$</td>
<td>Size of the candidate storage tank capacity indexed by $k$</td>
</tr>
<tr>
<td>$c_{ik}$</td>
<td>Capital cost for building a storage tank with capacity $C_k$ at node $i$</td>
</tr>
<tr>
<td>$c_{ijt}$</td>
<td>Operational cost for carrying unit flow from node $i$ to node $j$ at time period $t$</td>
</tr>
<tr>
<td>$d_{it}$</td>
<td>Demand for user node $i$ at time period $t$</td>
</tr>
<tr>
<td>$\mu_{ij}$</td>
<td>Arc $(i,j)$ loss factor</td>
</tr>
<tr>
<td>$P_{it}$</td>
<td>Pumping capacity for node $i$ at time period $t$</td>
</tr>
<tr>
<td>$RT_{it}$</td>
<td>Treatment/recharge capacity for node $i$ at time period $t$</td>
</tr>
<tr>
<td>$Q_{ijt}$</td>
<td>Pipeline capacity for arc $(i,j)$ at time period $t$ under normal operation and failure scenario, respectively</td>
</tr>
<tr>
<td>$\delta_{ijt}^f$</td>
<td>Failure indicator. If arc $(i,j)$ fails it is 1. Otherwise, 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{ik}$</td>
<td>Binary variable that takes 1 if a tank with capacity $C_k$ is built at noted $i$ at time period $t$, 0 otherwise</td>
</tr>
<tr>
<td>$q_{ijt}^o, q_{ijt}^f$</td>
<td>Flow on arc $(i,j)$ at time period $t$ under normal operation failure scenario, respectively</td>
</tr>
<tr>
<td>$V_{it}^o, V_{it}^f$</td>
<td>Storage for node $i$ at the end of time period $t$ under normal operation and failure scenario, respectively, $i \in ST$</td>
</tr>
<tr>
<td>$W_{it}^o, W_{it}^f$</td>
<td>Storage for node $i$ at the end of time period $t$ under normal operation and failure scenario, respectively, $i \in S$</td>
</tr>
<tr>
<td>$\theta_{itf}^f$</td>
<td>Water shortage for user node $i$ at time $t$</td>
</tr>
</tbody>
</table>
Table 2. Size and location of tank determined by the optimization model

<table>
<thead>
<tr>
<th>Pressure Zones</th>
<th>Tank Size (MG)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.6</td>
</tr>
<tr>
<td>D</td>
<td>0.4</td>
</tr>
<tr>
<td>E</td>
<td>0.4</td>
</tr>
<tr>
<td>FS</td>
<td>0.4</td>
</tr>
<tr>
<td>GS</td>
<td>0.4</td>
</tr>
<tr>
<td>HS</td>
<td>1.0</td>
</tr>
<tr>
<td>FN</td>
<td>0.4</td>
</tr>
<tr>
<td>GN</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Figure 1: Schematic of overall regional water supply system tri-level optimization model

Figure 2. RESIN study area layout (RESIN area is bounded by purple lines)
Figure 3. Schematics of the water supply system for the RESIN area (Darker pressure zones mean higher elevation zones).

Figure 4. Tank construction and operation costs determined by tri-level optimization.